MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. XXXV. *March* 12, 1875. No. 5.

PROFESSOR J. C. ADAMS, F.R.S., President, in the Chair.

Major-General J. C. Hannyngton, India Office, S.W.;
Prof. Robert Stawell Ball, F.R.S., M.R.I.A., University, Dublin;
H. Savile W. Evans, Esq., Wimbledon Park House;
A. M. W. Downing, Esq., B.A., Royal Observatory, Greenwich;
John Dreyer, Esq., M.A., Observatory, Parsonstown, Ireland;

were balloted for and duly elected Fellows of the Society.

On the Method to be used in Reducing the Observations of the Transit of Venus, 1874, *December* 8. By Sir G. B. Airy, K.C.B., Astronomer Royal.

In the Method of Reduction of the Observations of the late Transit of *Venus* which I now submit to the Royal Astronomical Society, there is no mathematical novelty. But the conditions of this series of observations are very different from those of any other observations known to me. Different nations and different observers are concerned in the observations; different theoretical views have animated the astronomers who have selected the stations, and planned the observations; instruments of different classes have been employed; different elements have been the subject of observation. I have, therefore, thought it well to call the attention of the Society to a form of reduction which will include in one group the observations made at all places, and by all methods; and in this form I hope the whole body of observations will be reduced, at least, in the first instance. It possesses this great value, amongst others, that it enables us at once to point out a faulty observation, which otherwise might lurk undiscovered, but producing a baneful effect on the conclusions. Those investigators, however, who desire to ascertain the inferences properly deducible from limited classes of observations, will find that this method gives them every facility. The equations to be used are ready to hand, and it is only necessary to separate them from the others. Still I hope that in the final reductions the whole mass of observations will be used in one group. Every

individual observation will then have an influence proportional to its merit ; and reason, as well as formula, leads us to think that the best final result will thus be obtained.

1. For the first step, we must call for the assistance of the computers of Ephemerides, or rather for the assistance of one computer of Ephemeris, as it is essential that the same elements and methods of computation of tabular places be employed throughout. The following elements must be computed with the utmost precision to $0''{\cdot}01$; second powers of time being used where necessary. It is not injurious to the result that these elements be wrong by a second or two ; but it is of the utmost importance that the same errors be continued throughout without the slightest alteration.

2. The cosmical elements upon which the phenomena of the Transit depend are—

The RA and NPD of the Sun's centre.
The Sun's angular radius.
The equatorial horizontal parallax of the Sun.
The RA and NPD of the centre of *Venus*.
The angular radius of *Venus*.
The equatorial horizontal parallax of *Venus*.

These elements ought to be prepared for intervals throughout the entire Transit, from a short time, say a minute, before the first external contact, as visible from the Earth's surface, to an equal time after the last external contact as visible from the Earth's surface. During the middle part of the Transit, when there is no question of close micrometer-measure or Janssen-measure, it may suffice to make these calculations for every minute of *sidereal* time ; but from the internal approach of limbs, say a minute or more from internal contact to a minute from external contact (or *vice versâ*, the minute-ranges being always interpreted in extension of the intervals here treated) the calculations ought to be made for every ten seconds of time.

3. To these elements are to be attached, where necessary, symbolical variations. We may assume the RA and NPD of the Sun's centre to be correct; any real error in them being afterwards applied to the similar elements for *Venus*. If our observations consisted merely of ocular observation of contacts, we might also assume the Sun's angular radius to be correct; as an error in it might be neutralised by errors of the RA and NPD of the centre of *Venus*. But an erroneous value of the Sun's radius would produce errors of different magnitudes in the measures of photographs made at different periods of the Transit (converted into sidereal measure by proportional parts of the Sun's photographic radius corrected for photographic irradiation), and therefore we must suppose the Sun's radius liable to error. This must be connected with corresponding errors of RA and NPD of *Venus*; and the Sun's angular radius for ocular observations (corrected for irradiation) must now be supposed liable

to the same error. Also, viewing the difference of the modes in which the disk of *Venus* will be treated, as affecting eye-observations of contact, and as affecting micrometer-observations or photographic records, we must consider the angular radius of *Venus* in contact-observations as liable to error. The RA and NPD of *Venus*, and both the parallaxes, are evidently to be taken as possibly erroneous. Thus we must use the following expressions, in which P and p are the numerical tabular values of the eq. hor. parallax of the Sun and *Venus*; p being considered $= n$. P, where n is the proportion of the distance of the Sun from the Earth to the distance of *Venus* from the Earth: R being the numerical tabular value of the angular radius of the Sun, and r that of *Venus*.

Angular radius of the Sun	=	R + δ.R.
Sun's equatorial horizontal parallax	=	P + δ.P.
Geocentric RA of the centre of *Venus*	=	Tabular ♀ RA + δ.♀ RA.
Geocentric NPD of the centre of *Venus*	=	Tabular ♀ NPD + δ.♀ NPD.
Angular radius of *Venus*, for contact-observations	=	$r + \delta.r$.
Equatorial horizontal parallax of *Venus*	=	$p + \delta.p$.
	=	n.P + $n.\delta$.P.

4. I proceed to point out the principal steps of the first computation in detail; partly for the purpose of indicating the various sources of the errors which ought to be taken into account in the computations conducted on the Theory of Probabilities; and partly to give facilities to the calculations of each computer by exhibiting in an orderly way the succession of steps of computation.

5. The transit-clock-time of each observation is to be taken, subject to error. And here it is to be observed that, where there are, at one place, multiple observations (as eye-observations both of ingress and of egress, or numerous micrometrical measures or photographic records), even though the probable errors of all the observations may be fairly considered equal, yet every individual observation must be considered as having its own independent Actual Error, up to the very last step of reduction where Probable Errors are introduced.

6. To this clock-time must be applied the correction of transit-clock, subject to error (probably so small that it may usually be neglected, but equally applicable to every one of multiple observations if it be retained). Thus the Local Sidereal Time of each separate observation is obtained.

7. The Terrestrial Longitude measured from the standard point of the Ephemeris is to be applied, subject to its possible error.

8. Thus is obtained, for each separate observation, the

Sidereal Time at the Ephemeris-standard-point; subject to the errors of transit-clock-time of observation, of transit-clock-correction, and of terrestrial longitude, and requiring corresponding corrections: expressed in terms of 1^s of Sidereal Time.

9. By use of these times there are to be computed for every observation,—

The Geocentric RA and NPD of the Sun's centre, affected by multiples of all the corrections mentioned above.

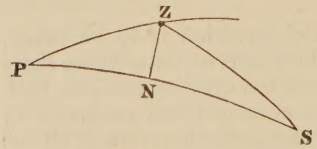
The Sun's angular radius (afterwards to be increased numerically by "augmentation," which is numerically sensible), affected by the symbolical correction δR.

The Sun's equatorial horizontal parallax (afterwards to be corrected numerically for geocentric radius of station), affected by the symbolical correction δ P.

The Geocentric RA and NPD of the centre of *Venus*, affected by multiples of all the corrections mentioned above, and also by the symbolical corrections δ. ♀ RA and δ. ♀ NPD.

10. Combining the Geocentric RA with the Local Sidereal Time in (6), the hour-angle is found. Symbolical corrections are not required, as this element is used only in computing small quantities (parallax in RA and in NPD). The Astronomical Latitude of Station is to be corrected by applying Angle of Centre so as to obtain Geocentric Latitude. The Eq. Hor. Parallax is to be multiplied by the proper elliptic factor to produce Local Horizontal Parallax. Then the following calculation is to be made.

11. In the diagram below, P is the pole above the horizon, Z the geocentric zenith, S the object (Sun's centre or *Venus*' centre), ZN an arc perpendicular to PS. Then,



$$\text{Sine ZN} = \text{Sine PZ} \,.\, \text{Sine } \angle \text{P}.$$

$$\text{Tan PN} = \text{Tan PZ} \,.\, \text{Cosine } \angle \text{P}.$$

$$\text{NS} = \text{PS} - \text{PN}.$$

$$\text{Factor of parallax in RA} = \frac{\text{Sine ZS} \times \text{Sine } \angle \text{S}}{\text{Sine PS}} = \frac{\text{Sine ZN}}{\text{Sine PS}};$$

$$\text{or} = \frac{\text{Sine PZ} \,.\, \text{Sine } \angle \text{P}}{\text{Sine PS}}.$$

$$\text{Factor of parallax in NPD} = \text{Sine ZS} \,.\, \cos \angle \text{S} = \text{sine ZS} \,.\, \tan \text{NS} \,.\, \cot \text{ZS}$$
$$= \cos \text{ZS} \,.\, \tan \text{NS} = \cos \text{ZN} \,.\, \text{sine NS}.$$

These factors should be computed separately for the Sun and for *Venus*.

12. Applying these parallaxes to the computed Geocentric Places, introducing all the symbolical corrections, and using the

symbol (n^r) to indicate the numerical value of the factor which in each place multiplies the symbolical corrections, we have the following Apparent Quantities :—

RA of Sun's Centre = Tabular ⊙ RA $+(n^r)\times\delta$. Terrestrial Longitude.
$+(n^r)\times$P $+(n^r)\times\delta$. Clock Time of Observation.
$+(n^r)\times\delta$.P.

NPD of Sun's Centre = Tabular ⊙ NPD $+(n^r)\times\delta$. Terrestrial Longitude.
$+(n^r)\times$P $+(n^r)\times\delta$. Clock Time of Observation.
$+(n^r)\times\delta$.P.

RA of *Venus*' Centre = Tabular ♀ RA $+\ \delta$.♀ RA.
$+(n^r)\times$P $+(n^r)\times\delta$. Terrestrial Longitude.
$+(n^r)\times\delta$. Clock Time of Observation
$+(n^r)\times\delta$.P.

NPD of *Venus*' Centre = Tabular ♀ NPD $+\ \delta$.♀ NPD.
$+(n^r)\times$P $+(n^r)\times\delta$. Terrestrial Longitude.
$+(n^r)\times\delta$. Clock Time of Observation
$+(n^r)\times\delta$.P.

where the symbol (n^r) has a different numerical value in every different place, excepting that for each element the values of (n^r) will be the same for P and δ.P, though differing from one element to another.

For the moment, we may use the following abbreviated expressions :—

Apparent RA of Sun's Centre = Tabular Apparent ⊙ RA + Δ.(⊙ RA),
Apparent NPD of Sun's Centre = Tabular Apparent ⊙ NPD + Δ.(⊙ NPD),
Apparent RA of *Venus*' Centre = Tabular Apparent ♀ RA + Δ.(♀ RA),
Apparent NPD of *Venus*' Centre = Tabular Apparent ♀ NPD + Δ.(♀ NPD),

where the term "Tabular Apparent" is used to denote "Tabular Geocentric Place affected by Tabular Parallax."

And these we shall further contract to

⊙ RA + Δ (⊙ RA),
⊙ NPD + Δ (⊙ NPD),
♀ RA + Δ (♀ RA),
♀ NPD + Δ (♀ NPD).

13. From these the Tabular Apparent Distance of the Centre of *Venus* from the Centre of the Sun will be computed by the following formula, of which the error is far below $0''\cdot01$:

$$\surd\{\sin(\odot\,\mathrm{NPD})\,.\,\sin\,\varphi(\mathrm{NPD})\,.\,(\odot\,\mathrm{RA}-\varphi\,\mathrm{RA})^2+(\odot\,\mathrm{NPD}-\varphi\,\mathrm{NPD})^2\}$$

(which line we may call Uncorrected Tabular Distance)

$$+\frac{\sin(\odot\,\mathrm{NPD})\,.\,\sin\,\varphi(\mathrm{NPD})\,.\,(\odot\,\mathrm{RA}-\varphi\,\mathrm{RA})}{\text{Uncorrected Tabular Distance}}\times\Big(\Delta(\odot\,\mathrm{RA})-\Delta(\varphi\,\mathrm{RA})\Big)$$

$$+\frac{\odot\,\mathrm{NPD}-\varphi\,\mathrm{NPD}}{\text{Uncorrected Tabular Distance}}\times\Big(\Delta(\odot\,\mathrm{NPD})-\Delta(\varphi\,\mathrm{NPD})\Big).$$

In these terms, $\odot\mathrm{RA}-\varphi\,\mathrm{RA}$ and $\odot\mathrm{NPD}-\varphi\,\mathrm{NPD}$ are to be expressed in arc, to the accuracy $0''{\cdot}01$.

The most convenient way of effecting the numerical calculation will be, by use of an auxiliary angle ϕ.

$$\text{Let } \tan\phi=\frac{(\odot\,\mathrm{RA}-\varphi\,\mathrm{RA})\times\surd\{\sin(\odot\,\mathrm{NPD})\times\sin(\varphi\,\mathrm{NPD}\}}{\odot\,\mathrm{NPD}-\varphi\,\mathrm{NPD}},$$

then the uncorrected tabular distance of centres is given by either of the following expressions :—

$$\frac{1}{\sin\phi}\times(\odot\,\mathrm{RA}-\varphi\,\mathrm{RA})\times\surd\{\sin(\odot\,\mathrm{NPD})\times\sin(\varphi\,\mathrm{NPD})\},$$

$$\frac{1}{\cos\phi}\times\{\odot\,\mathrm{NPD}-\varphi\,\mathrm{NPD}\};$$

and the corrected distance of centres is

$$\begin{aligned}&\text{uncorrected tabular distance of centres}\\&+\sin\phi\times\surd\{\sin(\odot\,\mathrm{NPD})\times\sin(\varphi\,\mathrm{NPD})\}\times\{\Delta(\odot\,\mathrm{RA})-\Delta(\varphi\,\mathrm{RA})\}\\&+\cos\phi\times\{\Delta(\odot\,\mathrm{NPD})-\Delta(\varphi\,\mathrm{NPD})\}.\end{aligned}$$

14. The expression just found is that which will apply when the subject of measure is the distance of centres of Sun and *Venus* at an observed time, as for instance in the measures of the distance of centres in photographs. It will be compared with the proportional part of the "Sun's augmented semidiameter affected with $\delta\mathrm{R}$", the proportion being taken from the measures of photographs, corrected as may be necessary. But when the phenomenon observed is the internal contact, the radius of *Venus* must be applied, and we have for corrected distance of the external limb of *Venus* from the Sun's centre,

$$\begin{aligned}&\text{uncorrected tabular distance of centres}+r\\&+\sin\phi\times\surd\{\sin(\odot\,\mathrm{NPD})\times\sin(\varphi\,\mathrm{NPD})\}\times\{\Delta(\odot\,\mathrm{RA})-\Delta(\varphi\,\mathrm{RA})\}\\&+\cos\phi\times\{\Delta(\odot\,\mathrm{NPD})-\Delta(\varphi\,\mathrm{NPD})\}\\&+\delta r.\end{aligned}$$

This will be compared with the Sun's augmented semidiameter affected with $\delta\mathrm{R}$.

15. In order to form the next combination of the equations given by the separate observations, we must be prepared to assign numerical values to the Probable Errors, or to the Probable Values of the Chance-Errors (of Local Time of Observation of the Phenomenon, or of Measure of the Photograph, as the case may be). For these estimations I do not think it is possible to lay down any mathematical rule. I think that each observer must give his careful attention to the circumstances, and must on his own judgment decide upon the value of the observed or measured element which he thinks best, and upon the range + or − which he thinks may be admitted as including half the possible errors. The measure of the half range (that is, from error = 0 to error = + value, or from error = 0 to error = − value) will be the Probable Error. We will now consider the application of this Probable Error under different circumstances.

16. First. Suppose that the observation has been the simple observation of internal contact, at a place whose longitude is accurately known. At that time, from the nature of the observation, the apparent distance of centres + the angular radius of *Venus* = Sun's augmented semidiameter. But the tabular calculation of apparent distance + angular radius of *Venus* gives a value (presumably) different from the Sun's augmented semidiameter; say, less by a quantity which we will call "Discordance." To remove this, we must apply corrections to the elements ♀RA, ♀NPD, R, r, P, and must convert all such quantities as those expressed by Δ in articles 13 and 14 into terms of $\delta.$♀RA, $\delta.$♀NPD, $\delta.$R, $\delta.r$, $\delta.$P; and must introduce a Chance-Error of Observed Time; so that,

$$\text{Sun's augmented semidiameter} + \delta.\text{R} =$$
$$= \left\{\begin{array}{l}\text{Value of tabular calculation of apparent distance of centres} + r \\ + (n^r) \times \delta.\text{♀RA} + (n^r) \times \delta.\text{♀NPD} + \delta.r + (n^r) \times \delta.\text{P} \\ + (\text{N}^r) \times \text{Chance-Error of Observed Time.}\end{array}\right\}.$$

Or, removing all but the last term to the left hand,

$$\text{Discordance} - (n^r) \times \delta.\text{♀RA} - (n^r) \times \delta.\text{♀NPD} + \delta.\text{R} - \delta.r - (n^r) \times \delta.\text{P} =$$
$$= (\text{N}^r) \times \text{Chance-Error of Observed Time.}$$

The corrections of ♀RA, ♀NPD, R, r, P, are, so far as they enter into this equation, definite quantities, free from chance-error; but the Observed Time has a Chance-Error; and has, therefore, a Probable Error, a definite quantity, to be determined as has been mentioned. Divide the equation by $(\text{N}^r) \times$ the Probable Error of Observed Time; and (remarking that, when a chance-error is divided by a definite quantity, its probable error is divided by the same definite quantity) we have

$$\frac{\text{Left Side of Equation}}{(\text{N}^r) \times \text{Probable Error of Observed Time}} = \frac{\text{Chance-Error of Observed Time}}{\text{Probable Error of Observed Time}}.$$

giving an equation in which the probable error of the second side =the abstract number 1. This is the state to which every observation must be reduced. The right side of the equation (which, if the observations were perfect, would = 0), is of no use, except as enabling us to say that the left side, as divided by $(N^r) \times$ Probable Error of Observed Time, is = 0 with Probable Error $= \pm 1$.

17. Second. Suppose that, with the ocular observation of internal contact, there are to be combined micrometer-observations, Janssen photographs, or any other measures, each of which alone can be interpreted as giving both an independent determination of the time of internal contact, and an estimate of the probable error of time of such independent determination. Put *p.e.* (1), *p.e.* (2), &c., for the probable errors of the ocular determination and the other independent determinations. Then, by a well-known theorem, the independent determinations are to be combined with the weights $\left(\frac{1}{p.e.\ (1)}\right)^2$, $\left(\frac{1}{p.e.\ (2)}\right)^2$, &c., to form the Observed Time for which all the calculations from the Ephemeris are to be made. And the quantity

$$\frac{1}{\text{Probable Error of Observed Time}} = \surd\left\{\left(\frac{1}{p.e.\ (1)}\right)^2 + \left(\frac{1}{p.e.\ (2)}\right)^2 + \text{&c.,}\right\}$$

which, with $\frac{1}{(N^r)}$, determines the factor of the left side of the equation.

18. Third. Suppose that the longitude of the station is not known with indisputable accuracy: and that observations are made similar to those of Articles 16 and 17. It will be remarked here that the correction of the longitude of the station, which is to be sought by independent means, is not a thing of the same class as the corrections of ♀ RA, P, &c., which are the special object of the observations of the Transit of *Venus*; nor is it a subject of observations of the moment, and liable to the accidental errors of the moment, like the observations of contacts, &c. Also, it may be expected that the longitude will in no long time be accurately determined. In this case then it will be proper, in preparing the terms on the left side of the equation, to attach to the clock-time of observation in Article 12 a symbolical correction $\delta.l$, and to combine its effect, in the shape of $(n^r) \times \delta.l$, with the "tabular calculation of apparent distances of centres" and therefore with the "Discordance." Then, in the final solution of equations, every correction δ. ♀ RA &c., δ.P &c., will contain a term multiplying $\delta.l$. And when the numerical value of $\delta.l$ shall be subsequently determined, the requisite alteration of the numerical values of δ. ♀ RA &c., δ. P &c., can then be effected at once.

The evaluation of the "Probable Error of Observed Time" is the same as when the longitude is certainly known.

These remarks apply to the treatment of all classes of observation, at every station where the longitude is uncertain.

19. Fourth. Suppose that observations are made at several stations, all of which depend for their longitude on one station, whose longitude is somewhat uncertain. In this case, the symbolical correction $\delta.l$ must be introduced into the calculations for each station; and then perhaps it will be simplest to exhibit separately the equations for each station. But they may be combined into one by using the combination-weights $\frac{1}{(\text{Probable Error of Observed Time})^2}$ proper for each: and that one resulting equation will contain a multiple of $\delta.l$.

20. Fifth. Hitherto we have confined our remarks to the observation of one phenomenon (Ingress or Egress), though with variations in the methods of observing and in the localities (supposed to be not widely separated). Now, suppose that at a station whose longitude is accurately known the Ingress and Egress are both observed.

As in Article 16, each phenomenon will give one equation; and, if the planet's geocentric place were subject to no error of RA or NPD, a favorable equation for Parallax would be obtained by taking the difference of these equations. But the errors of geocentric RA and NPD are sufficient to render this inaccurate. And although a great part of this inaccuracy would be removed by combination with a similar pair of equations under different terrestrial latitude, still there would remain a part, and the effects of different errors of observation might also interfere injuriously. It will be better to retain the equations uncombined.

21. Sixth. Suppose that Ingress and Egress are both observed at a station whose longitude is uncertain.

The inference is the same as in the last case. It will be best to retain the two equations given by the two phenomena, but both must contain the same longitude-correction $\delta.l$.

22. Seventh. Suppose that, during the progress of the Transit, a series of photographs is taken, at times that are subject to no sensible error except a possible error of longitude common to all; and suppose that measures of the photographs are taken, which give the "distances" of the centre of *Venus* from the centre of the Sun, in terms of the Sun's diameter on the photograph (corrected for photographic irradiation and for the effect of refraction on the apparent length of the inclined "distance," upon which corrections, however, we enter into no details here), subject to small independent errors in the measures or interpretations of the photographs. This case is different in some points from those previously considered. The changes in the relative positions of the Sun and *Venus* are so great that it is impossible to take a mean of the equations given by different photograph-measures. The element of uncertainty is not in time, but in measures. The equations must therefore be treated thus. The symbol for the correction of longitude $\delta.l$ must be kept on the left side of the equation, and multiplied in the successive operations by the proper factors, as in Articles 18, 19, 21. The chance-error of each measure must be kept on the right side, expressed

by the same unit (perhaps 1″·00) in which the computed measure of distances is expressed. The probable error of measure is to be expressed by the same unit. Then, on dividing by that probable error, an equation for each measure will be obtained whose probable error is = abstract 1, and which can be used in combination with all the other equations (produced by observations of a totally different kind) each of whose probable errors is = abstract 1.

23. The instances which have been given will probably suffice to guide the computer in forming the equations for any other combination of observations not described here. It remains to point out the steps of the well-known processes for ultimate treatment of the collected equations.

24. First, it is to be remarked that any theorist who is desirous of examining the validity of special combinations of stations (as, for instance, stations adapted to determination of result by difference of duration of Transit, or stations giving comparison of absolute times of the same phenomenon of Transit), or who desires to examine the results of special methods of observation (as, for instance, by photographs as distinguished from eye-observations), will find here all the materials ready to his hand. He has merely to extract the equations applying to his stations or method, and to combine them and treat them in the same manner as that in which the whole mass of equations are to be combined and treated. I earnestly hope, however, that the whole mass may be used in combination.

25. Next, as all the equations are so reduced as to be subject each to the probable error = abstract 1, the treatment is simple. Multiply every equation by its numerical coefficient of δ. RA of centre of *Venus* (omitting consideration of Probable Error), add together these multiplied equations, and one Final Equation is obtained. Multiply every equation by its numerical coefficient of δ. NPD of centre of *Venus*, add together the multiplied equations, and a second Final Equation is obtained. Similarly with regard to the numerical coefficients of δ. R, δ. Semidameter of *Venus*, δ. Horizontal Equatorial Parallax of the Sun. These five equations will give the five quantities sought. It will be remembered that every value derived from the equations will contain one term depending on each of the quantities δl.

26. The determination of probable error of final results, always troublesome, will perhaps be least troublesome in the following manner. In the right side of the equations which have been divided by Probable Error, instead of $\frac{\text{Actual Error}}{\text{Probable Error}}$ put a series of distinctive symbols, as (1), (2) &c., or any other convenient form; and on the left side, omit the "Discordance." Then combine and solve the equations precisely as for numerical determination of δ.*Venus*' RA, δ.*Venus*' NPD, &c., always using the numerical values on the left side, and the symbols (1), (2) &c., with proper numerical factors, on the right side. The solution will give

$$\delta.\textit{Venus}' \text{ RA} = A_1 \times (1) + A_2 \times (2) + \&c.$$
$$\delta.\textit{Venus}' \text{ NPD} = B_1 \times (1) + B_2 \times (2) + \&c.$$
$$\&c. \quad = \quad \&c.$$

where A_1, A_2 &c., B_1, B_2 &c., are numerical quantities. Then, as the probable error of each of the quantities (1), (2) &c., is = 1, we have

$$\text{Probable error of } \delta.\textit{Venus}' \text{ RA} = \sqrt{(A_1)^2 + (A_2)^2 + \&c.},$$

and so for the others.

The equations are solved by precisely the same process which has given the values of δ. *Venus*' RA &c.; and the factors (1), (2) &c., are abstract numbers. Hence, it is easily seen that the unit in the expressions for Probable Errors of δ.*Venus*' RA &c. is the same as the unit in the expressions for δ.*Venus*' RA &c., that is, as is most likely, 1″·00.

27. When the equations are solved (whether the entire collection, or separate groups as suggested in Article 24), the inferred values of $\delta.\varphi$ RA, $\delta.\varphi$ NPD, $\delta.R$, $\delta.r$, $\delta.P$, ought to be substituted in the equations of Articles 16 and 22, and the residual errors ought to be exhibited. The study of these residual errors will be peculiarly the work of the Superintending Astronomer, who shall be charged with the task of communicating to the world the important inferences deducible from the Transit of *Venus* 1874. The first of these, it is almost unnecessary to observe, is the value of Solar Parallax. The concomitant inferences, as to the corrections of the Tables of the Earth and *Venus*, will interest only the professional gravitational astronomer. But there will be other points, not of a numerical nature, but which are extracted most surely from numerical statements, to be ascertained from these comparisons:—The effect of the magnitude and excellence of the telescope for eye-views; the value and arrangement of the photographic system and pictures; the principles for the selection of stations; and other points which will present themselves to that scientific authority. The examination of these, it is to be hoped, would tend materially to an advance of accuracy in the results of the Transit of *Venus* 1882.

Finally, I would suggest for consideration the following points:—

(I). It is desirable that the Observations made in each of the National Expeditions should be printed with the least possible delay, under the superintendence of the head of the Expedition; and should be at once distributed to the principal scientific institutions and libraries, and to the persons officially interested in the Transit.

(II). It is desirable that these Observations should be printed in Quarto, of such size that all could be conveniently bound together. There is little difference in the sizes of the French

Memoirs, the Berlin Memoirs, the Smithsonian Memoirs, the Royal Astronomical Society's Memoirs; the size of the *Philosophical Transactions* is somewhat larger.

(III). A brief history of each Expedition should be given, with the names and offices of the persons employed; and description of the localities, accompanied by maps if necessary, and by such statements as may lead to identification of the localities of observations.

(IV). In the accounts of telegraphic operations, it may not be necessary to give details, if the instruments and methods are described, and a few observations are given sufficient for check of the fundamental conclusions; in the accounts of Transits, it is sufficient to give description of instruments and methods, constants of adjustments, and tables of clock-errors; and analogous abstracts of comparisons of chronometers, &c.

(V). But it is necessary to give in the fullest detail everything that bears upon the actual observation of contacts, or upon the observer's impression at the time of making the observation, or upon the micrometer-measures, or upon the photographs and the measures of the photographs, &c.; with sufficient description of the instruments and their adjustments at the time. Clock-time and Local Sidereal Time are to be given for every observation.

(VI). Reference must be made to the place of deposit of the original documents, observations, calculations, and instruments, of each National Expedition.

(VII). A large portion of the calculations sketched in the preceding Articles can be best made under the superintendence of the head of each National Expedition, and these should be printed and distributed as part of the Observations mentioned above.

(VIII). It is to be hoped that some astronomer of eminence may undertake to superintend, in some degree, the publications which I have suggested; and to make the final combination of observations and decide on the ultimate result. I can without difficulty fix on the foreign astronomer in whose hands I should be glad to see this work placed.

Royal Observatory, Greenwich,
1875, *Jan.* 19.

Report on the Progress made in the Calculations for a New Method of treating the Lunar Theory. By Sir G. B. Airy, K.C.B., Astronomer Royal.

In the *Monthly Notices* of this Society for 1874, January, I explained the form which I proposed to give to a New Method of treating the Lunar Theory, and I stated that I had actually commenced some of the numerical developments of which the new method mainly consists. In my usual Annual Report to the Board of Visitors of the Royal Observatory, I mentioned (in the ordinary course of report) that I had arranged such a method of treating the Theory, that the principal part of the work was purely numerical, and that a computer had been engaged, at my expense, in proceeding with some parts of the work. The Board spontaneously passed a resolution to the effect that this undertaking might properly be considered as a part or extension of the astronomical calculations made at the Observatory, and that in their opinion it might properly be undertaken at the public expense. I submitted this opinion of the Board of Visitors to the Admiralty, through whom it was placed before the Treasury; and, after explanation of the nature and object of the proposed work, I received authority from the Treasury for incurring the expense required for carrying on the computations through the current Financial year. I have subsequently submitted an estimate for progress of work during the year 1875–1876. Two computers have been employed, the aggregate of their times of employment to 31st December 1875 being about 10 months.

We have had painful evidence of the difficulties that may arise in utilising a laborious investigation, when by some sad chance the possibility of its completion by the original proposer has been suddenly terminated, and scarcely sufficient explanation has been left to enable successors to enter into the remaining steps in the same spirit as that which guided the primary operations. It has appeared to me, therefore, desirable in prosecuting a new theory (which, though far simpler than preceding theories in the intellectual labour demanded, is still sufficiently extensive and sufficiently novel in form to require some study before a following astronomer could actively take it up), to place on record such a description of the method of proceeding, and, from time to time, such a statement of the advance of the work, as will enable a successor, if necessary, to carry on the operations without much difficulty to their close.

1. Referring to my paper "Proposed New Method &c." already cited, paragraph marked VI., it will be seen that the first step is to convert Delaunay's sexagesimal angles, expressed in seconds of arc, into equivalent expressions in decimal fractions

of radius; that these expressions are to be limited to the seventh decimal place, of which the unit is 10^{-7}; and that, this limit being understood throughout, trouble and confusion will be avoided by omitting decimal cyphers preceding the effective figures. To convert the seconds of arc into decimals of radius, it is necessary to multiply the number of seconds by 0·000004848137. Therefore, multiplying by 100000, and considering the final figure of Delaunay's coefficients to be the second decimal of the sexagesimal second, we have to arrange a method, applicable to all coefficients, by which 10000000 may be converted into 4848137. The method adopted is one analogous to that known in books of arithmetic by the term "Practice;" it never leaves a doubt on the value of the leading figure, and it employs no unnecessary figures. By trial, the following steps are adapted for this case; every succeeding number being formed from the preceding number by use of the fraction in the margin:

		10000000	
$+\frac{1}{2}$	=	+ 5000000	
$-\frac{3}{100}$	=		− 150000
$-\frac{1}{90}$	=		− 1667
$-\frac{1}{8}$	=		− 208
$+\frac{1}{20}$	=	+ 10	
$+\frac{1}{5}$	=	+ 2	
		+ 5000012	− 151875
		− 151875	
		+ 4848137	

In the application to actual coefficients, the last term is never wanted, and the preceding term only once. Of the numbers, which I now proceed to exhibit as specimens, the first is the largest coefficient in longitude, the second is one taken from the middle of the series:—

		22639″·06				36″·16	
$+\frac{1}{2}$	=	+1131953		$+\frac{1}{2}$	=	+1808	
$-\frac{3}{100}$	=		−33959	$-\frac{3}{100}$	=		−54
$-\frac{1}{90}$	=		− 377	$-\frac{1}{90}$	=		− 1
$-\frac{1}{8}$	=		− 47			+1808	−55
						− 55	
$+\frac{1}{20}$	=	+ 2				+1753	
		+1131955	−34383				
		− 34383					
		+1097572					

In succeeding parts of the work, several thousands of multiplications are performed in the same manner; the numbers to be multiplied being arranged in series, of which every term has the same multiplier; and the marginal fractions for that multiplier being determined by trial.

In some cases it is necessary to perform division; thus, in order to reduce the expression for parallax from the form

$$\text{Parallax} = 3422''{\cdot}700 + \text{series of terms}$$

to the form

$$\frac{a}{r} = \{1 + \text{new series of terms}\},$$

it is necessary to determine by trial a process which will convert 34227000 into 10000000. The marginal fractions are found to be

$$+\frac{1}{3},\ -\frac{1}{9},\ -\frac{1}{10},\ -\frac{1}{9},\ -\frac{1}{30}.$$

2. The number of coefficients of cosines of different arguments in the expression for Parallax, adopted from Delaunay, and converted by the process above described, is 100. The number of coefficients of sines of different arguments in the expression for Longitude, is 170; and that in the expression for Latitude, is 142; which are similarly converted. By subsequent operations, the numbers of coefficients in squares, &c., of these series are raised to 148, 170, 180. The smallest value retained is 1 (in my notation), or a number not less than its half.

3. The great mass of operations consists in multiplying together two series, such as

$$A + B\cos\boxed{P} + C\cos\boxed{Q} + D\cos\boxed{R} + \text{etc.}$$

$$a + b\cos\boxed{p} + c\cos\boxed{q} + d\cos\boxed{r} + \text{etc.}$$

The coefficients of the upper line are all multiplied by a (by the process above described); then by b, and so on. The general product is

$$Aa + Ba\cos\boxed{P} + Ca\cos\boxed{Q} + \text{etc.} + Ab\cos\boxed{p} + \frac{Bb}{2}\cos\boxed{P+p} + \frac{Bb}{2}\cos\boxed{P-p}$$
$$+ \frac{Cb}{2}\cos\boxed{Q+p} + \frac{Cb}{2}\cos\boxed{Q-p} + \text{etc.} + Ac\cos\boxed{q} + \frac{Bc}{2}\cos\boxed{P+q}$$
$$+ \frac{Bc}{2}\cos\boxed{P-q} + \frac{Cc}{2}\cos\boxed{Q+q} + \frac{Cc}{2}\cos\boxed{Q-q} + \text{etc.}$$

In the great majority of instances, the new arguments formed in these multiplications are to be found in each of the series which are multiplied together; and the form of the series produced by the collection of these products is the same as that of the first series, with the addition, perhaps, of a few small terms. In the primary series for Parallax, Longitude, and Latitude, the coefficients are arranged in the order of descending magnitude, and this order is little disturbed in the operations of multiplying series together.

4. The coefficients of $\frac{a}{r} - 1$ are represented by the ordinal numbers (written in red ink) 1, 2, 3, &c., to 100; those of $\left(\frac{a}{r} - 1\right)^2$ by 201, 202, &c., to 348, the small figures following centuries being attached to the same cosines, as far as they go: those of $\left(\frac{a}{r} - 1\right)^3$ by 401, 402, &c., to 535, the small figures following centuries being attached to the same cosines, &c. The coefficients of latitude are represented by 2401, 2402, &c., those of longitude by 5001, 5002, &c.; and the coefficients of products and other functions formed from them, by different series of similar order, the first always being a century increased by 1.

5. When powers of $\frac{a}{r} - 1$, to the fifth power, have been formed by successive multiplications by the series for $\frac{a}{r} - 1$, (forming a new series for each power, of which the terms are distinguished by new centuries of red-ink numbers), all the required powers of $\frac{a}{r}$ are formed by the binomial formula:

$$1 + n\left(\frac{a}{r} - 1\right) + \frac{n(n-1)}{2} \cdot \left(\frac{a}{r} - 1\right)^2 + \text{etc.}$$

When powers of Latitude have been formed by successive multiplication by the series for Latitude, the expression for sine $\lceil \text{Latitude} \rceil$ and the expressions for the various powers of cosine $\lceil \text{Latitude} \rceil$ are formed by means of the appropriate formulæ in powers of Latitude.

6. For the formation of $\frac{dv}{dt}$, it is to be remarked that every argument is formed by the sum of multiples of four simple arguments, namely,

D = Excess of Moon's mean longitude above Sun's mean longitude.

f = Moon's mean argument of latitude.

l = Moon's mean anomaly.

L = Sun's mean anomaly.

The proportion of the daily increase of each of these arguments to the daily increase of the Moon's mean longitude is taken as adopted or found by Delaunay, and the unit of time is supposed to be such that the Moon's mean motion in longitude = 1. Thus, for the term

$$\sin \lceil 2\,\mathrm{D} + 2f + l - \mathrm{L} \rceil,$$

the sum of the proportions of daily increases of terms under the bracket to daily increase of Moon's mean longitude is found to be 4·7751; therefore

$$\frac{d}{dt} \sin \lceil 2\,\mathrm{D} + 2f + l - \mathrm{L} \rceil$$

$$= \cos \lceil 2\,\mathrm{D} + 2f + l - \mathrm{L} \rceil \times 4{\cdot}7751 \times \frac{d}{dt}(\text{daily increase of Moon's mean longitude})$$

$$= 4{\cdot}7751 \times \cos \lceil 2\,\mathrm{D} + 2f + l - \mathrm{L} \rceil.$$

The corresponding term in $\frac{dv}{dt}$ is found by multiplying this quantity by the coefficient of the corresponding term in v. Succeeding differentiations are effected on the same principle.

7. The methods explained to this point apply to the expansion of functions on the basis of elements assumed by Delaunay, as representing the Moon's motion at a standard modern year (say 1900); without any consideration of the corrections which are due to the coefficients in those expansions, in order to produce a correct theory, on the supposition that the elements of the Earth's orbit round the Sun are stable; and without any consideration of the corrections which are due to the slow changes of the

elements of the Earth's orbit, which produce secular terms in the Moon's motions. Notice of these corrections will be taken hereafter. At present I may with propriety record the progress made in the limited expansions to which I have first alluded.

8. The following quantities (each expanded into a series of numerous terms with numerical coefficients and periodical trigonometrical functions) are computed, all in duplicate, and some of the earliest in triplicate.

Subject of Expansion.	Red-ink Nos.
Modified equivalent of	
$\frac{a}{r}$ and $\frac{a}{r} - 1$	1— 100
$\left(\frac{a}{r} - 1\right)^2$	201— 348
$\left(\frac{a}{r} - 1\right)^3$	401— 531
$\left(\frac{a}{r} - 1\right)^4$	601— 725
$\left(\frac{a}{r} - 1\right)^5$	801— 826
$\left(\frac{r}{a}\right)^4$	1001—1148
$\left(\frac{r}{a}\right)^3$	1201—1348
$\left(\frac{r}{a}\right)^2$	1401—1548
$\frac{r}{a}$	1601—1748
$\frac{a}{r}$ (above)	1— 100
$\left(\frac{a}{r}\right)^2$	1801—1948
$\left(\frac{a}{r}\right)^3$	2001—2148
$\left(\frac{a}{r}\right)^4$	2201—2348
Equivalent for Latitude	2401—2542
(Latitude)^2	2601—2726
(Latitude)^3	2801—2959
(Latitude)^4	3001—3128
(Latitude)^5	3201—3246

Subject of Expansion.	Red-ink Nos.
(Latitude)^6	3401—3434
sin $\lceil 1 \rceil$	3601—3754
cos $\lceil 1 \rceil$	3801—3927
$(\cos \lceil 1 \rceil)^2$	4001—4127
$(\cos \lceil 1 \rceil)^3$	4201—4327
$(\cos \lceil 1 \rceil)^4$	4401—4527
$\cos \lceil 1 \rceil \cdot \frac{r}{a}$	4601—4790
$\left(\cos \lceil 1 \rceil \cdot \frac{r}{a}\right)^2$	4801—4995
Equivalent for Longitude	5001—5182
$\frac{dv}{dt}$	5201—5382
$\frac{dv}{dt} \cdot \left(\cos \lceil 1 \rceil \cdot \frac{r}{a}\right)^2$	5401—5645
$\left(\frac{dv}{dt}\right)^2 \cdot \left(\cos \lceil 1 \rceil \cdot \frac{r}{a}\right)^2$	5701—5945
$\frac{d}{dt}\left(\cos \lceil 1 \rceil \cdot \frac{r}{a}\right)$	6001—6191
$\left\{\frac{d}{dt}\left(\cos \lceil 1 \rceil \cdot \frac{r}{a}\right)\right\}^2$	6201—6450
(Velocity)^2, or sum of last quantity + last but two	6501—6750
$\frac{d}{dt}$(Velocity)^2	6801—7050

(The velocity here mentioned is the resolved velocity parallel to the plane of the Earth's orbit.)

The duplicate calculation of the quantity last recorded is not yet completed.

The calculation of the following quantities is not yet begun :—

$$\left(\frac{a}{r}\right)^3 \cdot \frac{d}{dt}\left\{\left(\cos \lceil 1 \rceil \cdot \frac{r}{a}\right)\right\},$$

$$\frac{r}{a} \cdot \sin \lceil 1 \rceil,$$

$$\frac{d^2}{dt^2}\left(\frac{r}{a} \cdot \sin \lceil 1 \rceil\right),$$

$$\left(\frac{a}{r}\right)^2 \cdot \sin \lceil 1 \rceil \cdot$$

These complete the list of all the quantities not multiplied by the disturbing force which enter into the Equations of the Lunar Theory.

Some of the computed numbers, applying to equations of long period, may require further revision.

9. I now proceed to remark on the state of the "symbolical corrections" mentioned in article (2*) of my former paper, on which the corrections of Delaunay's coefficients, as depending on the elements of the Earth's orbit in a given year (say 1900), are based; and from which the secular equations of the Moon's motion, depending on the slow changes of those elements, are to be deduced. For the present, I limit my details to the symbolical corrections of the coefficients of the periodic terms. The notation used to express these symbolical variations is, the red-ink number defining the coefficient (as above described) surrounded by a ring. This applies to every coefficient in the first instance, and to every subsequent coefficient, except where, by virtue of multiplication by small numbers, it is made too small to possess any value. Thus the series for $\frac{a}{r} - 1$ is expressed by

$$\left(\text{red-ink } 2 + \textcircled{2}\right) . \cos \lceil l \rceil + \left(\text{red-ink } 3 + \textcircled{3}\right) . \cos \lceil 2\,\mathrm{D} - l \rceil + \text{etc.};$$

the series for the longitude-terms added to mean longitude is

$$\left(\text{red-ink } 5002 + \underline{\textcircled{5002}}\right) . \sin \lceil l \rceil + \left(\text{red-ink } 5003 + \underline{\textcircled{5003}}\right) . \sin \lceil 2\,\mathrm{D} - l \rceil + \text{etc.};$$

the series for latitude is

$$\left(\text{red-ink } 2401 + \underline{\textcircled{2401}}\right) . \sin \lceil f \rceil + \left(\text{red-ink } 2402 + \underline{\textcircled{2402}}\right) . \sin \lceil f + l \rceil + \text{etc.},$$

where "red-ink 2," "red-ink 3," "red-ink 5002," "red-ink 2401," &c., are numerical quantities, adopted or computed as above-mentioned; but

$$\textcircled{2},\ \textcircled{3},\ \underline{\textcircled{5002}},\ \underline{\textcircled{2401}},\ \text{etc.}$$

are still preserved in the symbolic form.

10. As the further treatment of these symbolical terms tends to increase their number and the complexity of their combinations, it is necessary from the beginning to adopt some rule as to the anticipated magnitude of the corrections which Delaunay's coefficients may require. Now I think I may safely assume that no correction greater than $2''{\cdot}00$, or 100 of my notation, can result from the investigation. In this case, it is useless to retain any product by a number smaller than $0{\cdot}01$, or (with allowance for turning of the figure) smaller than $0{\cdot}005$, or smaller than

50000 of my notation. In the improbable case of an error ultimately showing itself with magnitude greater than 2″·00, the only effect of our limitation would be, that in the equations there is occasionally a loss of some quantity amounting numerically to 1 in the last figure.

11. The application of these considerations will be best understood from an example. The first instance of multiplication is that of the formation of

$$\left(\frac{a}{r}-1\right)^2 \text{ by multiplying } \left(\frac{a}{r}-1\right) \text{ by } \left(\frac{a}{r}-1\right);$$

or

$$\left(r.i.\ 2+\textcircled{2}\right).\cos\lceil l\rceil + \left(r.i.\ 3+\textcircled{3}\right).\cos\lceil 2\,\mathrm{D}-l\rceil + \left(r.i.\ 4+\textcircled{4}\right).\cos\lceil 2\,\mathrm{D}\rceil + \left(r.i.\ 5+\textcircled{5}\right).\cos\lceil 2\,l\rceil + \text{etc.};$$

multiplied by

$$\left(r.i.\ 2+\textcircled{2}\right).\cos\lceil l\rceil + \left(r.i.\ 3+\textcircled{3}\right).\cos\lceil 2\,\mathrm{D}-l\rceil + \left(r.i.\ 4+\textcircled{4}\right).\cos\lceil 2\,\mathrm{D}\rceil + \left(r.i.\ 5+\textcircled{5}\right).\cos\lceil 2\,l\rceil + \text{etc.},$$

which will produce

$$\left(r.i.\ 201+\overline{(201)}\right).\cos\lceil 0\rceil + \left(r.i.\ 202+\overline{(202)}\right).\cos\lceil l\rceil + \left(r.i.\ 203+\overline{(203)}\right).\cos\lceil 2\,\mathrm{D}-l\rceil + \left(r.i.\ 204+\overline{(204)}\right).\cos\lceil 2\,\mathrm{D}\rceil + \left(r.i.\ 205+\overline{(205)}\right).\cos\lceil 2\,l\rceil + \text{etc.}$$

Now, $\cos\lceil 0\rceil$ is produced by the combination of $\cos\lceil l\rceil$ with $\cos\lceil l\rceil$, of $\cos\lceil 2\,\mathrm{D}-l\rceil$ with $\cos\lceil 2\,\mathrm{D}-l\rceil$, of $\cos\lceil 2\,\mathrm{D}\rceil$ with $\cos\lceil 2\,\mathrm{D}\rceil$, of $\cos\lceil 2\,l\rceil$ with $\cos\lceil 2\,l\rceil$, etc., and its coefficient, or $r.i.\ 201+\overline{(201)}$, will therefore be

$$\begin{aligned}
&\tfrac{1}{2}\,(r.i.\ 2)^2 + r.i.\ 2\times\textcircled{2},\\
&+\tfrac{1}{2}\,(r.i.\ 3)^2 + r.i.\ 3\times\textcircled{3},\\
&+\tfrac{1}{2}\,(r.i.\ 4)^2 + r.i.\ 4\times\textcircled{4},\\
&+\tfrac{1}{2}\,(r.i.\ 5)^2 + r.i.\ 5\times\textcircled{5},\\
&+\text{etc.}
\end{aligned}$$

Now $r.i.\ 2 = +545094$; $r.i.\ 3 = +99812$; $r.i.\ 4 = -82321$;

r.i. 5 = +29794. Here then it appears that *r.i.* 2, *r.i.* 3, *r.i.* 4, are above our limit; but *r.i.* 5 and all succeeding numbers (which are still smaller) are below it. The first three, then, of the products of symbolical variations just given, are the only ones to be retained for (201). Striking off the last five figures we have

$$(201) = +5 \times (2) + 1 \times (3) + 1 \times (4),$$

where the unit of the factors is $\frac{\text{unity}}{100}$, or 10^{-2}.

Now consider the factor of cos $\lceil l \rceil$ in the term

$$\left(r.i.\ 202 + (202)\right) . \cos \lceil l \rceil .$$

Here, cos $\lceil l \rceil$ may be produced by the combination of cos $\lceil l \rceil$ with cos $\lceil 2l \rceil$, of cos $\lceil 2\,D - l \rceil$ with cos $\lceil 2\,D \rceil$, of cos $\lceil 2\,D \rceil$ with cos $\lceil 2\,D - l \rceil$, of cos $\lceil 2\,D - l \rceil$ with cos $\lceil 2\,D - 2l \rceil$, and of others which are at once seen to be insensible. Hence, the value of $(r.i.\ 202 + (202))$ is

$$+ \left(r.i.\ 2 + (2)\right) \times \left(r.i.\ 5 + (5)\right),$$

$$+ \left(r.i.\ 3 + (3)\right) \times \left(r.i.\ 4 + (4)\right),$$

$$+ \left(r.i.\ 4 + (4)\right) \times \left(r.i.\ 6 + (6)\right),$$

$$+ \left(r.i.\ 3 + (3)\right) \times \left(r.i.\ 19 + (19)\right);$$

and the value of (202) is

$$+ r.i.\ 2 \times (5) \qquad + r.i.\ 5 \times (2),$$

$$+ r.i.\ 3 \times (4) \qquad + r.i.\ 4 \times (3),$$

$$+ r.i.\ 4 \times (6) \qquad + r.i.\ 6 \times (4),$$

$$+ r.i.\ 3 \times (19) \qquad + r.i.\ 19 \times (3).$$

The only factors which are sensible are *r. i.* 2, *r. i.* 3, *r. i.* 4; and therefore

$$(\overline{202}) = +5 \times (5) + 1 \times (4) + 1 \times (3) + 1 \times (6) + 1 \times (19),$$

where (as before) the unit of the factors is $\frac{\text{unity}}{100}$, or 10^{-2}.

When the series for $\left(\frac{a}{r} - 1\right)^2$ is multiplied by another series, a similar process must be used for expressing the symbolical variations of the coefficients then newly formed, in terms of $(\overline{201})$, $(\overline{202})$, etc.; and of the variations of terms of the other series; then the values found above are to be substituted for $(\overline{201})$, $(\overline{202})$, etc., and finally the symbolical variation of every coefficient, without exception, will be found in terms of

(2), (3), (4), etc., $(\overline{2401})$, $(\overline{2402})$, etc., and $(\overline{5001})$, $(\overline{5002})$, etc.

12. The symbolical variations are formed as far as

$$\left(\cos \overline{|l|} \times \frac{r}{a}\right)^2$$

in the table of article 8, but may require further revision.

13. To complete the exhibition of symbolical variations, it is necessary to prepare the variations produced by the symbolical variations of the arguments. Thus, for the term

$$r.i.\ 8 \times \cos \overline{|2\,D - l - L|},$$

the complete variation is

$$(8).\cos \overline{|2\,D - l - L|} - r.i.\ 8.\sin \overline{|2\,D - l - L|} \times (\overline{2D - l - L}).$$

In the ordinary process of multiplying series by series, this operation is remarkably simple; it is not necessary to go step by step, as in the preparation of the variations of the coefficients in article 11; but the variation depending on the argument will be found correctly from the argument standing at the end of the operations (as will easily be seen by trying a step-by-step operation). A trifling complexity is introduced in these terms which bear the sign of differentiation, because the symbolical variations themselves are subject to differentiation.

14. As the variation of arguments produces its effect in the grand final equation only where it is multiplied by a coefficient which is usually small, and as it is particularly in these variations that we must look for the terms which rise to importance in the shape of secular terms, we must be prepared to accept much larger values for the magnitude of the terms to be produced by these symbolical variations of arguments. I have not yet decided on the limit to be adopted.

15. No step of calculation has yet been made in reference to the symbolical variation of arguments.

16. No calculation is yet made for that side of the final equation which contains the disturbing force as a factor.

In any future Report which it may appear desirable to place before the Society, there will be no occasion to repeat any part of the explanation of methods given here, and it may be anticipated that the statement of the step which the operations may have reached will be very brief.

Royal Observatory, Greenwich,
1875, *February* 6.

Observations of the Satellites of the Planet Uranus, made at Birr Castle, during the years 1872, 1873, *and* 1874.

By the Earl of Rosse.

The following observations of the Satellites of the planet *Uranus*, made with the reflector of six-feet aperture at Birr Castle, though for several reasons not as satisfactory as might be desired and might be obtained with that instrument, may perhaps be of some value as a contribution to knowledge in a department of astronomy in which so few have as yet worked.

The shortness of the time (40 minutes) available each night for the observation of the planet with the 6-foot instrument, the atmospheric disturbance so often a source of annoyance in using so large an aperture, the unsuitability of the micrometer for the work, and to a certain extent also the almost unavoidable deterioration of the polish of the mirrors in the course of working, which probably increased the diffused light round the planet, combined to detract from the accuracy of the observations, and to render the detection of the inner satellites difficult.

My late assistant, Dr. Ralph Copeland, has compared the observed places of *Oberon* and *Titania* with those which he has calculated from Dr. von Asten's definite circular Elements given in Vol. xviii., No. 5, page 26, of the *Memoirs* of the Impérial Academy of Sciences of St. Petersburg. The differences between calculation and observation are given in the column headed (C.—O.). The observations made by Dr. Copeland and myself are distinguished respectively by the letter C. or R.

Oberon.

Date.	Sid. Time. h m	Position.	No. of Measures.	Distance.	No. of Measures.	C.−O. Position.	C.−O. Distance.	Observer.
		°		″		°	″	
1872 Jan. 13	8 20	324·9	2	...	...	−1·3	...	C.
,, ,, 15	8 14	259·0	2	...	...	−0·6	...	C.
,, ,, 18	8 15	179·3	1	...	...	+1·3	...	C.
,, ,, 19	8 26	161·9	2	...	...	−1·4	...	C.
,, Feb. 3	8 16	122·5	4	...	...	−0·6	...	C.
,, ,, 5	8 9	53·4	4	...	...	0·0	...	R.
,, ,, 6	8 17	30·3	3	...	...	−3·8	...	R.
,, ,, 7	8 23	4·8	5	...	...	+0·4	...	C.
,, ,, 10	8 24	294·3	3	...	...	−0·8	...	R.
,, ,, 12	8 19	226·4	2	...	...	−0·6	...	R.
,, Mar. 2	8 19	89·9	10	...	...	−2·6	...	C.
1873 Jan. 16	8 40	174·3	2	...	...	−1·2	...	C.
,, ,, ,,	8 48	...	...	44·93	2	...	+0·29	C.
,, Feb. 6	8 42	343·0	1	43·81	1	−5·2	−1·49	R. & C.
,, ,, 19	8 35	350·0	3	...	...	−2·3	...	C.
,, ,, ,,	8 32	...	3	41·98	2	...	+2·60	C.
,, ,, 20	8 44	325·5	2	36·16	2	+0·1	+2·95	C.
1874 Jan. 16	8 56	144·6	1	35·49	1	−6·6	+3·01	C.
,, ,, 20	9 3	18·4	2	43·47	2	−2·6	+1·24	C.
,, Feb. 3	8 46	9·8	2	44·70	2	−2·3	+1·43	C.
,, ,, 4	8 49	352·8	2	44·80	2	−2·0	−0·34	C.
,, ,, 18	9 4	340·7	1	40·83	1	0·0	+0·62	R.
,, ,, 21	9 3	235·9	1	...	...	+1·9	...	R.
,, ,, ,,	9 5	...	...	36·47	1	...	−4·23	R.
,, Mar. 7	9 4	221·9	3	...	...	−0·4	...	R.
,, ,, ,,	9 2	...	...	35·03	2	...	+0·18	R.
,, ,, 18	8 57	316·3	2	34·55	2	−1·2	−0·95	C.

Titania.

1872	Jan.	13	8 26	39·2	1	...	....	−2·6	...	C.
,,	,,	15	8 16	334·6	2	...	...	−6·9	...	C.
,,	,,	18	8 12	191·2	1	...	...	+3·3	...	C.
,,	,,	19	8 26	165·4	2	...	...	−1·7	...	C.
,,	Feb.	3	8 21	257·6	5	...	...	−0·8	...	C.
,,	,,	5	8 9	176·8	4	...	...	+1·0	...	R.
,,	,,	6	8 20	139·6	4	...	...	+5·0	...	R.

Date.			Sid. Time. h m	Position. °	No. of Measures.	Distance. "	No. of Measures.	C.−O. Position. °	C.−O. Distance. "	Observer.
1874	Feb.	7	8 21	97·9	4	...	...	−0·7	...	C.
,,	,,	10	8 24	339·1	2	...	...	−1·6	...	R.
,,	,,	12	8 14	245·0	3	...	...	−2·4	...	R.
,,	Mar.	2	8 16	184·1	10	...	...	−1·0	...	C.
1873	Jan.	16	8 40	272·6	2	...	...	−5·5	...	C.
,,	,,	,,	8 47	...	...	22·64	2	...	+0·66	C.
,,	Feb.	6	8 42	138·1	3	...	...	−5·0	...	R. & C.
,,	,,	,,	8 43	...	...	25·55	2	...	+1·66	...
,,	,,	19	8 36	318·8	3	[29·12]	[1]	−1·7	[−1·09]	C.
,,	,,	20	8 47	263·5	2	18·35	2	−1·5	+5·55	C.
1874	Jan.	16	8 59	309·9	1	24·29	1	−1·8	−0·74	C.
,,	,,	20	9 10	148·8	1	25·69	1	−3·6	+1·36	C.
,,	Feb.	3	8 46	278·4	2	20·90	2	−4·3	+0·52	C.
,,	,,	4	8 52	217·4	2	25·17	2	+1·1	+3·03	C.
,,	,,	18	9 2	12·9	1	31·58	1	−2·5	+3·07	R.
,,	,,	21	8 55	239·4	1	29·11	1	+0·6	−5·13	R.
,,	Mar.	7	8 59	25·6	3	31·68	3	−2·2	+0·05	R.
,,	,,	18	8 55	314·6	2	24·87	2	−2·3	+0·11	C.
				Umbriel.						
1872	Feb.	6	8 32	302·9	1	...	...	...	...	R.
1873	,,	19	8 46	176·5	2	17·82	2	...	...	C.
				Ariel.						
1872	Feb.	6	8 28	217·3	1	...	...	...	...	R.
1873	Jan.	16	8 40	197·6	2	...	...	...	...	C.
,,	,,	,,	8 46	...	...	15·38	1	...	...	C.

Notes and Remarks.

1872. Jan. 13. The Satellites very difficult to measure. The diameter of *Uranus* estimated equal to the thickness of the position-wire found = 3″·26 or thereabouts.

,, ,, 18. Sky hazy; speculum in a state of strain.

,, Feb. 3. *Oberon* far brighter than *Titania*. Power 414.

,, ,, 10. All four Satellites seen on the same side of the planet, but only *Oberon* and *Titania* measured.

1873. Jan. 16. Definition at times good. No trace of any markings seen. *Ariel* very difficult to measure, especially in distance. Diameter of *Uranus* = 5″·29 (2 measures).

1873 Feb. 19. There are two accordant measures of *Titania*, giving the distance = 22″·87 and one making it 29″·12. Probably the former measures are both wrong to the extent of 10 parts of the micrometer-head = 6″·511.
„ „ 20. *Titania* excessively faint; sky hazy.
1874. Jan. 16. *Titania* seen only after some examination. Definition very inferior, but power 625 shows the Satellites better than 414.
„ Feb. 3. Moon shining into mouth of tube.
„ „ 4. Clouds drifting over, but Satellites seen fairly at times. Moon troublesome.
„ „ 18. At 9h. 7m. a faint fixed Star ? was measured. Position 255°·0; Distance 51″·12.
„ Mar. 7. *Oberon* rather fainter than *Titania*.
„ „ 18. A 14-mag. Star 23°·7, 80″·70.

Dr. Copeland remarks that the negative sign predominates in the column headed (C. — O.), and hence our observations of the position-angle deviate from Herr Struve's on the same side as Dr. von Asten has found those of M. Lamont and Messrs. Lassell and Marth to do. In fact, von Asten gives at page 25 the following differences :—

	Oberon.	Titania.
Struve — Lamont	— 1°·78	— 1°·91
Struve — Lassell (1852)	— 1·26	— 2·03
Struve — Marth	— 1·77	— 1·92

and Dr. Copeland finds—

Struve — Rosse	— 1·38 (60 obs.)	— 1·29 (61 obs.)

Great care was taken in the determination of the zero of the position-circle of the micrometer, which was obtained by setting the wires parallel to the diurnal motion of the stars, the telescope being fixed, and referring their position to that of the wires of a collimator firmly fixed inside the tube, of the construction described by Sir J. Herschel.*

Where the night was not calm and otherwise favourable reference to the collimator-wires rather than directly to the equator was adopted, as the more reliable method. The position-angles have all been corrected for the error due to the peculiar arrangement of the axis on which the telescope turns.†

A magnifying power of 414 was generally used, but occasionally the inner satellites could be better seen with a power of 625.

Dr. Copeland found values for the semi-axis major less than those of Struve by 0″·654 for *Oberon* and 1″·389 for *Titania*; but as in the micrometer the distances were measured by coincidence of the apparent centres of the objects with the *internal* edges of

* See *Outlines of Astronomy*, Art. 204 *f.*; also, *Phil. Trans.* 1833, &c.
† See *Phil. Trans.*, 1861, p. 700.

opaque bars without illumination, it would be hardly possible to avoid systematic error in measuring distances, and thence to arrive at any *very* accurate result as to the mass of the planet; but I may say that $\frac{1}{23577}$ from observations of *Oberon*, and $\frac{1}{24422}$ from observations of *Titania*, are the values he has deduced for the mass.

1875, *March* 1.

Note on the Colour of Uranus. By Captain W. Noble.

Turning back to the records of observations made by me at various and unequal intervals during the last eighteen years, I find my visual impressions of the tint exhibited by the disk of *Uranus* always embodied in the words, "a pale blue disk."

On the night of Tuesday last, however (March 9), on turning my telescope on to the planet at $9^h\ 45^m$ L.M.T., I was at once struck with the apparent alteration in the character of its light, which was much *whiter*, and brighter than I ever remember to have seen it. Nay, there was even the faintest suspicion of *yellow* in it; so thoroughly had every vestige of blueness vanished.

The weather has prevented me from repeating my observation; but I lose no time in publishing it in order that other observers, with superior optical means to my own, may either confirm, or disprove my suspicion.

We are all familiar, by this time, with the remarkable change of colour exhibited by *Jupiter* at intervals; and it would be a curious coincidence were a cognate variation found to exist in this more distant member of our system.

I had *Uranus* under observation from $9^h\ 45^m$ until about $10^h\ 10^m$ L.M.T. with powers of 154 and 255 on my 4·2 in.-Ross Equatoreal; and the change of tint from that which I recollect appeared as marked with the higher power as with the lower one.

Forest Lodge, Maresfield,
1875, *March* 12.

Ephemeris for Physical Observations of Mars. By A. Marth, Esq.

(*Communicated by Mr. De La Rue.*)

Opposition of 1875.

Greenwich, Noon. 1875.	Angle of Posit. of Mars' Axis.	Areographical W. Long. of the Centre of Mars' Disk.	Diff.	Lat.	Diameter.	Amount of Greatest Defect of Illumination.	Posit.	Areocentric Ang. between Sun & Earth.
	°	°		°	″	″	°	°
May 1	26·93	140·7		−5·53	13·87	1·03	269·48	31·64
			1753·6					
6	26·46	94·3		5·71	14·66	0·97	269·44	29·81
			53·9					
11	26·13	48·2		5·74	15·49	·89	269·63	27 96
			54·1					
16	25·95	2·3		5·63	16·34	·79	270·10	25·37
			54·4					
21	25·94	316·7		5·35	17·21	·66	270·94	22·66
			54·7					
26	26·09	271·4		4·92	18·01	·53	272·3	19·64
			54 9					
31	26·42	226·3		4·33	18·92	·38	274·4	16·28
			55·1					
June 5	26·93	18·14		−3·59	19·70	·24	277·9	12·64
			1755·4					
10	27·57	136·8		2·73	20·38	·12	284·4	8·78
			55·6					
15	28·32	92·4		1·79	20·92	·04	300·5	4·99
			55·7					
20	29·13	48·1		−0·81	21·30	·01	3·0	2·77
			55·7					
25	29·95	3·8		+0·15	21·50	·05	57·8	5·49
			55·7					
30	30·74	319·5		1·06	21·51	·15	73·3	9·49
			55·6					
July 5	31·44	275·1		+1·86	21·34	·30	79·6	13·64
			1755·3					
10	32·00	230·4		2·50	21·00	·49	83·2	17·65
			55·2					
15	32·42	185·6		2·96	20·52	·71	85·6	21·41
			54·9					
20	32·68	140·5		3·23	19.95	0·92	87·23	24·87
			54·6					
25	32·76	95·1		3·29	19·31	1·13	88·40	28·03
			54·3					
30	32·68	49·4		3·17	18·63	1·32	89·19	30·86
			54·0					
Aug. 4	32·43	3·4		+2·85	17·93	1·48	89·74	33·38
			1753·7					
9	32·02	317·1		2·37	17·23	1·61	89·87	35.58
			53·5					

Greenwich, Noon. 1875.	Angle of Posit. of Mars' Axis.	Areographical W. Long. of the Centre of Mars' Disk.	Diff.	Lat.	Diameter.	Amount of Greatest Defect of Illumination.		Areocentric Ang. between Sun & Earth.
	°	°		°	″	″	°	°
Aug. 14	31·45	270·6		1·72	16·54	1·71	89·86	37·51
			1753·2					
19	30·73	223·8		0·93	15·87	1·78	89·65	39·17
			53·1					
24	29·87	176·9		+0·02	15·23	1·83	89·26	40·60
			52·8					
29	28·87	129·7		−1·01	14·62	1·86	88·72	41·82
			52·6					
Sept. 3	27·73	82·3		−2·13	14·03	1·87	88·06	42·85
			1752·4					
8	26·46	34·7		−3·34	13·48	1·87	87·27	43·71

June 7. Equinox, the Sun passing from the North to the South side of the Equator.

Areographical Longitude and Latitude of the Centre of the Disk and Apparent Diameter at the Times to which the Sketches in the Publications of the Royal Astronomical Society refer.

Opposition of 1862.

Memoirs of the Royal Astronomical Society, Vol. xxxii.

Mr. Lockyer. Plates I.–IV.

		Date	h m		°	°	″
Pl. I.	1	1862 Sept. 23	10 25	G.M.T.	350·2	−21·3	22·9
	2		11 55		12·1	...	...
	3		12 55		26·7	...	...
	4	17	10 50		49·4	−20·7	22·5
II.	1		after				
	2	Oct. 18	8		94·3	−24·2	21·4
	3	15	9 8		137·5	−23·9	21·8
	4	rather past	9 20		140·5	...	...
III.	1	Oct. 11	11 4		201·2	−23·4	22·3
	2	9	10 47		214·7	−23·1	22·5
	3	3	10 30		263·3	−22·4	22·9
	4		11 23		276·2	...	...
IV.	1		11 51		283·0	...	...
	2	Sept. 25	10 44		337·2	−21·5	...
	3		10 50		338·6	...	...
	4	23	9 40		339·2	−21·3	...

Lord Rosse. Plate V.

	Date	h m		°	°	″
Pl. V. 1	1862 July 22	22 30	Birr Castle Sid. Time	278·0	−23·2	14·6
2	Sept. 14	0 26		113·5	−20·5	22·2

Assuming that, the hour 6^h given on the engraving being obviously a mistake, the substitution of $0^h\ 26^m$ is the most plausible conjecture.

3	Sept. 16	23 55	86·2	−20·6	22·4
4	Oct. 6	2 10	283·3	−22·8	22·7
5	29	1	39·5	−25·3	19·5
6	Nov. 6	1 40	328·9	−25·9	17·9

Mr. Lassell. Plates VI., VII.

Pl. VI. 1	1862 Sept. 13·439 G.M.T.		81	−20·5	22·1
2	20·44		19	−21·0	22·7
3	22·44		1	−21·2	22·8
4	24·44		343	−21·4	22·9
5	25·44		335	−21·5	22·9
6	27·46		320	−21·7	23·0
7	29·43		296	−21·9	23·0
8	Oct. 11·40		180	−23·4	22·3
9	13·40		162	−23·7	22·0
10	15·40		144	−23·9	21·8
11	17·43		137	−24·1	21·5
12	18·43		128	−24·2	21·4
VII. 13	1862 Oct. 21·36		77	−24·6	20·9
14	23·39		69	−24·8	20·5
15	25·38		48	−25·0	20·2
16	27·31		6	−25·2	19·8
17	Nov. 4·32		297	−25·8	18·3
18	5·32		288	−25·9	18·1
19	15·30		189	−26·3	16·3
20	17·28		163	−26·4	15·8
21	18·28		154	−26·4	15·7
22	22·29		120	−26·5	15·0
23	Dec. 8·29		329	−26·3	12·5
24	11·29		300	−26·1	12·1

Opposition of 1864.

Mr. Dawes. *Monthly Notices*, Vol. xxv.; *Memoirs*, Vol. xxxiv.

					h m		°	°	″
No.	2	1864	Nov.	3	12 24	G.M.T.	127·5	− 1·2	16·4
	4		Dec.'	1	12 0		233·7	− 6·6	17·3
	6		Nov.	26	11 46		274·3	− 5·6	17·4
	8			20	11 36		324·6	− 4·2	17·4
	10			14	12 0		24·0	− 3·0	17·2
	12			12	12 30		49·1	− 2·6	17·1
	13			10	12 6		60·5	− 2·3	17·0
	15	1865	Jan.	21	8 0		75·1	−10·1	11·2

Opposition of 1873.

Mr. Knobel. *Monthly Notices*, Vol. xxxiii. p. 476 ff.

No.	1	1873	Apr.	20	11 40	G.M.T.	269·0	+16·8	16·0
	2			23	11 10		235·4	+17·3	16·2
	3				12 15		251·2	...	...
	4			26	11 20		211·5	+17·9	16·4
	5		May	11	9 15		49·5	+20·4	...
	6				10 0		60·4	...	...
	7			12	7 45		18·7	+20·5	...
	8				12 10		83·3	+20·6	...
	9			13	12 20		76·9	+20·7	16·3
	10			16	11 0		30 9	+21·1	16·2
	11			19	10 20		354·6	+21·5	16·0
	12				12 30		26·2	...	...
	13			22	11 30		344·9	+21·9	15·8
	14			23	9 0		299·5	+22·0	15·7
	15			30	9 50		248·7	+22·6	15·1
	16			31	9 30		234·7	+22·6	15·0
	17		June	8	8 30		147·3	+22·9	14·2

It is very desirable that, when sketches are made, the times should be properly noted, when the leading points are laid down.

Photography in the Transit of Venus. By W. de W. Abney, R.E.

I venture to offer a few remarks on a subject which has lately seemed to occupy the minds of a limited portion of the press correspondents who write on astronomical subjects, I mean on the supposed inferiority of the English method of solar photography in comparison with the American during the late Transit of *Venus*. Much stress has been laid on the fact that you can measure the focus of a 40-foot telescope and calculate the diameter of the Sun as it should appear on the plate. It seems to me that there is no more difficulty in calculating the diameter with the English photoheliograph than with this. At all events, it is not an impossibility. This difficulty being placed on one side, there seems to me to be very little superiority left for the American system, and, further, there are drawbacks to it which cannot be glossed over. Supposing that the images be distorted in both systems, in the English system that distortion can only arise from causes in the telescope itself, and can be corrected by photographing a scale, in different relative positions to the object-glass. This is what is being done at Greenwich now. Distortion in the American system can also arise from the heliostat; and I think that the difficulty of estimating the amount due to the telescope itself, as well as to the heliostat, will be a matter for the gravest consideration. We are also told that irradiation is a great drawback to our system. I think the effect of irradiation has been vastly overrated. The indistinctness which often accompanies the Sun's limb is very little due to it in my opinion. It is principally caused by the heating of the atmosphere between ourselves and the Sun, and the consequent disturbance apparent only round its edge. The irradiation in the plates we used is reduced to a minimum, as the albuminate of silver which fastens the minute particles of iodide and bromide of silver together is nearly of the same specific gravity as that of the two latter. Experiment after experiment has shown to me that irradiation proper is really due to a reflexion from these minute particles, and if we connect them, forming nearly a continuous mass of the same specific gravity, it must be materially reduced, and perhaps eliminated. The irradiation due to reflexion from the back of the plate can be totally avoided; and I have found no difference in using glass as a vehicle in which to carry the collodion, or a substance totally opaque, when the plates were prepared as we used them.

Daguerreotype plates are not free from irradiation, as is claimed for them, as can be proved by experiment; and in my humble judgment it was most judicious of the Astronomer Royal not to have insisted on their employment, as they are subject to much greater fluctuations of expansion than are glass negatives. They are much less sensitive, and it would have been almost impossible to have produced a 4-inch picture of the Sun with a moderate

exposure. Indistinctness of the Sun's limb is more apparent with wet plates than with dry, for a reason I should like to state. In dry plates the principal development is carried out by the alkaline method, which only utilizes the bromide in the plate, whilst with wet plates the iodide *and* bromide are both utilized. The sensitiveness of iodide to weak radiations is much greater than of bromide, hence the atmospheric disturbance round the Sun's limb is developed to a less distance with the former than with the latter, there being a rapid diminution in the intensity of the light, as the distance from the limb is increased. On but few days is the atmosphere undisturbed; when it is, the limb will be nearly as sharp by one method as by the other, though the dry plate will still have an advantage due to the lessened irradiation. One great disadvantage in the American system is this, that there is a body of air heated between the heliostat and the object-glass. This must cause an unequal seething round the Sun's limb, and an error in measurements might occur from this alone.

It may be of interest to some to learn this fact, viz., that at Rodriguez 9 Janssen plates were exposed, and not 9 images on one Janssen plate. In a letter in one daily paper I read that as each Janssen plate was to contain 60 pictures, 51 failed, as there were only 9 exposed. The writer fell into an error here, mistaking the word "plate" for something else. These accidental slips often prove most injurious, and should be corrected. I have therefore taken this opportunity of saying that at Rodriguez the Janssen plates were quite successful, and that the proper number of pictures were obtained in each of the 9, making a total of 441.

Note on the Transit of Venus. By J. E. Gore, Esq.

(*Communicated by Mr. Browning.*)

I saw the transit of *Venus* with the telescope you sent me (a 3-in. achromatic). I could see no black-drop at ingress, but at egress, using a reduced aperture of $1\frac{1}{2}$ in., a very small ligament was visible. At ingress with the full aperture, I distinctly saw the disk of *Venus* outside the Sun's limb sometime before internal (external?) contact.

Umballa, India,
1875, *January* 25.

Venus on the Sun's Limb at the Transit of 1874. By E. J. Stone, M.A., F.R.S.

I forward a copy of a photograph of *Venus* on the Sun's limb, taken at Cape Town, by Mr. Harvey, assistant to Mr. Bruton.

I wish to call attention to the evident distortion of the image of *Venus*. It is clearly elongated in the direction of the Sun's diameter, instead of being shortened, as might have been expected. There is no such distortion shown in the photographs taken before *Venus* and the Sun's limb appeared in contact.

If we admit the evidence of a sensible irradiational enlargement of the Sun's diameter and contraction of the diameter of *Venus* when seen projected upon the Sun's disk, the observed distortion of form is at once accounted for. I must confess my inability to account for the observed facts, except upon the assumed existence of irradiation to a sensible extent.

There is one point to which I should wish to recall the attention of those who enter into discussions of the supposed subjective character of the "black-drop" or "ligament" phenomenon.

If the "black-drop" arises from irradiation it is restricted to the angular limits of the irradiation produced under the circumstances of vision, and it must be useless to expect its appearance in a very feebly illuminated field.

Note on the Transit of Venus. By Admiral Ommanney, C.B.

(*Extract from a letter to E. Dunkin, Esq.*)

With reference to the times of contact which I reported to you in my letter from Luxor, I must inform you that after making that communication, a more accurate determination of the chronometer-error was arrived at by my associates, Professors Auwers and Döllen, which has been adopted, as you will perceive, in the report I submitted to the Astronomer Royal; therefore, instead of the times given in the *Monthly Notices* of January last, the corrected time of observations should stand thus:

	h	m	s
Time of internal contact at egress	20	16	4·5
,, last external contact at egress	20	44	27·0

which leaves the interval the same as before, $28^{m}\ 22^{s}\cdot5$.

1875, *March* 10.

Accounts of the Observations of the Transit of Venus have been received from:

1. R. J. L. Ellery, Esq., Director of the Melbourne Observatory. The observations were made at—

The Melbourne Observatory. The weather was overcast, but the internal contact at ingress and the two contacts at egress were observed.

Mornington, on the shore of Port Phillip Bay, about 30 miles S. The weather was cloudy, but internal contact at ingress and two contacts at egress were observed. Prof. Wilson, who had charge of the observations died about two days after the transit, on December 11.

View Hill, Sandhurst, about 86 miles N.W. Weather bad; internal contact at egress observed.

Glen Rowan, about 150 miles N.E. Weather bad; internal contact at ingress observed.

2. H. C. Russell, Esq., Director of the Sydney Observatory. The stations were:

Sydney Observatory. All four contacts observed, and 180 pictures taken, forming a beautiful record of the whole transit.

Eden, Twofold Bay. Cloudy weather; the two contacts at ingress observed; 40 photographs taken.

Goulburn, a town on the mountains 2,300 feet above the sea. Internal contact at ingress and the two contacts at egress. No photographs, owing to an accident.

Woodford, the private residence of A. Fairfax, Esq., about 54 miles from Sydney on the Western Railway. All four contacts observed, and 63 whole pictures taken, and 16 Janssen plates with 60 partial photographs on each: some very interesting as bearing on the black-drop.

Mr. Allerding also observed at Sydney on his own account, and Messrs. Belfield and Park at Armidale, near Sydney.

3. J. Tebbutt, Esq., Windsor, New South Wales. Weather very favourable, and all four contacts observed.

4. E. J. Stone, Esq., Astronomer Royal for the Cape of Good Hope. The weather was good, and the first and second contacts at egress were observed.

5. Capt. A. C. Bigg-Wither, at Mooltan, Punjáb, India. The weather was good. Sun rose with planet about half way on disk: internal contact observed, and the two contacts at egress.

These will be published, together with the Accounts of the Observations of the Government Expeditions, in a volume of the *Memoirs*.

Additional Observations of Comet III, 1874 (*Coggia's*).

By John Tebbutt, Esq.

The following are the last of the series of observations of Coggia's Comet. The differential results are uncorrected for refraction and parallax, the former correction being scarcely appreciable. The comet was excessively faint and much diffused during October, so that the observations for that period will probably be of little value.

1874.	Windsor M.T. h m s	Comet's App. R.A.	m s	Comet's App. N.P.D.	′ ″	No. of Comps.
Aug. 29	17 16 25	B.A.C. 2796	+0 54·89	B.A.C. 2796	−11 20·2	8
Sept. 1	16 44 15	Lacaille 3296	−0 13·22	Lacaille 3296	− 7 58·1	11
„ 7	7 39 20	*a*	−4 39·90	*a*	− 0 44·7	1
„ 8	7 40 22	*b*	−4 50·97	*b*	− 1 55·8	4
„ 14	16 24 28	Lacaille 3202	+9 6·48	Lacaille 3202	+ 1 27·4	2
„ 14	16 34 36	Lacaille 3254	+4 8·03	Lacaille 3254	− 1 1·6	7
„ 21	11 45 56	*c*	+0 55·72	*c*	+ 2 31·5	10
„ 28	7 44 56	*d*	−4 33·99	*d*	+ 0 17·1	1
Oct. 2	9 51 38	*e*	+0 42·62	*e*	+ 9 24·6	2
„ 6	9 57 36	Lacaille 2927	+5 31·90	Lacaille 2927	− 9 7·7	2
„ 6	9 57 36	Lacaille 3027	−3 46·87	... ...	...	2
„ 7	10 23 58	Lacaille 2927	+2 7·35	... ...	...	1
„ 7	10 23 58	Lacaille 3027	−7 9·92	Lacaille 3027	+ 9 15·4	1

The following are the approximate positions of the unknown comparison-stars with reference to stars which could be identified at the time.

Comp.-Star.	Mag.	R.A.	m s	N.P.D.	″
a	8	B.A.C. 2863	− 4 33·5	B.A.C. 2863	+12 47
b	8	B.A.C. 2863	− 4 30·3	B.A.C. 2863	+35 48
c	8	B.A.C. 2835	−14 17·2	B.A.C. 2835	−17 10
d	$7\frac{3}{4}$	B.A.C. 2726	− 0 9·0	B.A.C. 2726	+ 2 44
e	$7\frac{1}{2}$	Lacaille 2961	+12 5·1	Lacaille 2961	− 4 8
		Lacaille 3010	+ 8 12·2	Lacaille 3010	− 2 32

Windsor, New South Wales,
1875, *January* 7.

Observations of the Eclipse of the Sun, 1874, *April* 16, *at the Royal Observatory, Cape of Good Hope.*

Observer.	Instrument.	Power employed.	Cape Mean Time. First contact. h m s	Cape Mean Time. Last contact h m s
Mr. Finlay	8½-feet Equatoreal	200	2 38 0·5	4 55 37·7
„ Maclear	46-inch „	90	2 38 8·3	...
„ Stevens	Common telescope	60	2 38 6·5	4 55 39·3
„ Freeman	Comet-seeker	15	2 38 32·5	4 55 37·2
„ Black	Seaman's glass	25	2 38 30·3	4 55 40·5
„ Lowrie	Eye	...	2 39 0·3	4 55 27·8
„ Shoobin, (A Russian officer attached to H.I.M.S. "Vsadnix," then in Table Bay.)	Theodolite	20	2 38 20·6	4 55 12·2

Mr. Black considers his first contact quite 10 seconds late.

Discovery of Minor Planet (143).

M. Palisa discovered at Pola, on the evening of February 23, another member of the group of minor planets. It was also observed at Berlin on the following evening. The positions determined from these first observations are as follow:—

			Mean Time at place of observation. h m s	R.A. h m s	N.P.D. ° ′ ″
Pola	1875	Feb. 23	8 42 0	3 57 56	76 14 0
Berlin	„	„ 24	8 27 14	3 56 57·7	76 13 20

The planet was of the twelfth magnitude.

On the Application of Corrections for Change of Temperature to the Rates of Chronometers at Sea. By Mr. Hartnup.

The *Monthly Notices* for December, 1874, contains a letter from Mr. A. E. Nevins, an officer of the ship "Tenasserim," on the performance of two chronometers during a voyage from Liverpool to Calcutta. From some oversight the name of Mr. Nevins does not appear in that letter. On the return voyage of the "Tenasserim" from Calcutta to London, the chronometers were compared with each other, and the temperature taken daily in the same way as on the voyage out; and through the courtesy of Captain Potts, Mr. Nevins has been able to supply me with

a continuation of his calculations: so that we now have the difference of Greenwich Mean Time between the two chronometers by calculation and by comparison with each other for every fifth day—with two or three unimportant exceptions—during a period of eight months.

It must not be assumed that the two chronometers of the "Tenasserim" are inferior instruments, or that they are more imperfect than usual in thermal adjustment; on the contrary, they are much better instruments, and more perfect in thermal adjustment than the average of those we meet with in the Merchant Navy. This being the case, it is difficult to conceive how a greater improvement could be made in the use of the chronometer at sea than that of adopting a variable rate dependent on the temperature, instead of one uniform rate throughout a long voyage in variable climates. In an observatory furnished with the necessary means there is no difficulty in supplying the rates for the temperatures between 50° and 95°; and as it appears to be the practice at sea to compare chronometers with each other daily, surely the mariner, if supplied with the means, might be induced to take the rates corresponding to the temperatures from a record similar to the following, which was used on the voyage of the "Tenasserim" from Calcutta to London:—

Table of Rates for every 5° of Temperature.

Chronometer	50°	55°	60°	65°	70°	75°	80°	85°	90°	95°
	s	s	s	s	s	s	s	s	s	s
No. 209	−1·8	−1·2	−0·6	−0·1	+0·4	+0·9	+1·3	+1·7	+2·1	+2·4
No. 713	−1·3	−0·8	−0·5	−0·4	−0·4	−0·6	−0·9	−1·4	−2·1	−3·0

By taking the rates from the above table the calculated difference between the two chronometers differed from the observed by only $1^m\ 18^s{\cdot}5$ after a voyage of eight months; but by using the rates as found in Liverpool in 55°, the difference between observation and calculation amounted to $10^m\ 16^s{\cdot}0$, as shown in the accompanying table.

The following were the errors of the two chronometers when the ship arrived in London:—

	No. 209.				No. 713.			
		h	m	s		h	m	s
From G. M. T., obtained in London .	Fast	0	2	3·0	Slow	0	36	20·0
By calculation from rates corrected for change of Temperature . .	„	0	1	58·0	„	0	35	6·5
Differences, or errors of Longitude, by Chronometers . . .	„	0	0	5·0	„	0	1	13·5

The following continuation of Mr. Nevins' Table exhibits the difference of Greenwich Mean Time between the two chronometers as obtained from calculation and comparison with each other during the voyage from Calcutta to London.

Date.	Errors on G. M. T., from calculations with rates corrected for change of Temperature.		Difference between the two Chronometers from		Difference of G. M. T. between the two Chronometers from rates		Mean Temperature Fahrenheit.
			Calculation.	Comparison with each other.	Corrected for change of Temperature.	Uncorrected for change of Temperature.	
1874	No. 209.	No. 713.					
	m s	m s	m s	m s	m s	m s	°
May 31	+0 28·5	−33 21·0	33 49·5	33 51·5	0 2·0	4 52·5	88
June 3	+0 35·1	−33 27·9	34 3·0	34 0·0	0 3·0	5 2·2	91
7	+0 42·7	−33 35·1	34 17·8	34 10·5	0 7·3	5 14·3	88
11	+0 50·3	−33 41·9	34 32·2	34 23·5	0 8·7	5 28·9	87
18	+1 2·2	−33 51·7	34 53·9	34 48·0	0 5·9	5 56·2	85
25	+1 13·4	−34 0·8	35 14·2	35 16·5	0 2·3	6 27·5	84
30	+1 20·9	−34 6·3	35 27·2	35 39·0	0 11·8	6 52·0	82
July 5	+1 28·0	−34 11·4	35 39·4	35 59·5	0 20·1	7 14·5	81
10	+1 35·0	−34 16·5	35 51·5	36 19·5	0 28·0	7 36·5	81
15	+1 41·0	−34 20·5	36 1·5	36 38·5	0 37·0	7 57·5	79
20	+1 44 5	−34 23·0	36 7·5	36 54·5	0 47·0	8 15·5	73
25	+1 46·0	−34 25·0	36 11·0	37 6·0	0 55·0	8 29·0	69
30	+1 46·5	−34 27·0	36 13·5	37 13·0	0 59·5	8 38·0	67
Aug. 4	+1 45·5	−34 29·0	36 14·5	37 17·5	1 3·0	8 44·5	64
9	+1 46·0	−34 31·0	36 17·0	37 22·0	1 5·0	8 51·0	67
14	+1 42·5	−34 34·0	36 16·5	37 21·0	1 4·5	8 52·0	59
19	+1 38·5	−34 37·0	36 15·5	37 18·0	1 2·5	8 51·0	58
24	+1 37·5	−34 39·0	36 16·5	37 17·0	1 0·5	8 52·0	64
29	+1 38·0	−34 41·0	36 19·0	37 19·0	1 0·0	8 56·0	67
Sept. 3	+1 41·5	−34 43·5	36 25·0	37 24·5	0 59·5	9 3·5	73
8	+1 46·5	−34 47·0	36 33·5	37 34·0	1 0·5	9 15·0	76
13	+1 53·0	−34 51·5	36 44·5	37 46·5	1 2·0	9 29·5	80
18	+1 58·5	−34 55·5	36 54·0	38 1·0	1 7·0	9 46·0	78
23	+2 2·5	−34 58·0	37 0·5	38 13·0	1 12·5	10 0·0	74
28	+2 6·5	−35 0·5	37 7·0	38 23·0	1 16·0	10 12·0	74
Oct. 3	+2 3·5	−35 3·0	37 6·5	...	...	...	60
8	+1 58·0	−35 6·5	37 4·5	38 23·0	1 18·5	10 16·0	56

For the preceding errors and rates—

The sign + indicates fast, or gaining.

The sign − indicates slow, or loosing.

Errata.

Monthly Notices—December 1874.

P. 79, in 55°, 70°, 85°, $r\ r'\ r''$ should be in seconds of time.

,, 81, line 2—for *ratio* read *rates*.

,, ,, under Table of Rates—for 95° read 92°.